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Different Client, Same Deal

Multiplying and Dividing Rational Expressions

LEARNING GOALS

In this lesson, you will:

- Multiply rational expressions.
- Divide rational expressions.

Imagine that 6 adults order a 12-slice pizza for delivery. Simple division should tell you that each adult gets 2 slices in order to divide the pizza fairly. Consider a situation where a parent brings home a big bag of marbles for her 3 children. If the bag contains 108 marbles, the simple number equation $108 \div 3 = 36$ allows the parent to determine how to fairly divide the marbles up so that each of her children is happy. Have you ever thought that this type of division may not always be the best way to fairly divide up resources? Consider the following problem:

Five animals share the resources in a wooded region of land. They all eat fruit, berries, and nuts, and must acquire an adequate amount of food to get them through the winter. They live on several acres of land that contains exactly 50 pounds of food. How much food should each animal receive? Does your answer change when you learn that the five animals are a squirrel, a mouse, a bird, a deer, and a bear? How would you divide the 50 pounds of food so that each animal gets a "fair" amount? Does it make sense that they all receive the same amount of resources? Is dividing 50 by 5 a fair way to go about this problem?

The mathematical concept "fair division" is an interesting mathematical concept. What criteria do you think should be used to determine whether resources are divided fairly?

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PROBLEM 1 Take Advantage of the Clearance on Expressions

Previously, you learned that adding and subtracting rational expressions involved the same process as adding and subtracting rational numbers. Now, you will see that multiplying rational expressions involves the same steps as multiplying rational numbers.

Remember that when you multiply rational numbers you can simplify at the beginning or the end, and the result is the same value; however, simplifying earlier saves time and will keep the numbers smaller.

1. Consider Method A compared to Method B in both columns of the table.

	Rational Numbers	Rational Expressions Involving Variables
Method A	$\frac{2}{15} \cdot \frac{5}{8} = \frac{10}{120}$ $= \frac{1}{12}$	$\frac{2x}{15x^2} \cdot \frac{5x^2}{8} = \frac{10x^3}{120x^2}$ $= \frac{1x}{12}$
Method B	$\frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}} = \frac{1}{12}$	$\frac{\overset{1}{\cancel{2}}x}{\underset{3}{\cancel{15x^2}}} \cdot \frac{\cancel{5x^2}}{\underset{4}{\cancel{8}}} = \frac{1x}{12}$

a. Explain the difference in the methods.

b. Which method do you prefer?

2. Brody says $x \neq 0$ for the equation in the table, $\frac{2x}{15x^2} \cdot \frac{5x^2}{8} = \frac{1x}{12}$. Damiere says that there are no restrictions because the answer is $\frac{x}{12}$ and there are no variables in the denominator. Who is correct? Explain your reasoning.

3. Analyze Isha's work.

Isha

$$\frac{12xyz^2}{11} \cdot \frac{33x}{8z} = \frac{\overset{9}{\cancel{396}}\, x^2 y \overset{z}{\cancel{z^2}}}{\underset{2}{\cancel{88}}\,\cancel{z}}$$

$$= \frac{9x^2yz}{2}$$

Explain how Isha could have multiplied the rational expressions more efficiently.

4. Shaheen multiplies $\frac{5x^2}{3x^2 - 75} \cdot \frac{3x - 15}{4x^2}$ without simplifying first.

Shaheen

$$\frac{5x^2}{3x^2 - 75} \cdot \frac{3x - 15}{4x^2} = \frac{15x^3 - 75x^2}{12x^4 - 300x^2}$$

$$= \frac{15x^2(x - 5)}{3x^2(4x^2 - 100)}$$

$$= \frac{\overset{5}{\cancel{15x^2}}(x - 5)}{\underset{1}{\cancel{3x^2}}(4x^2 - 100)}$$

$$= \frac{5(x - 5)}{4(x^2 - 25)}$$

$$= \frac{5\cancel{(x - 5)}}{4\cancel{(x - 5)}(x + 5)}$$

$$= \frac{5}{4(x + 5)}$$

Complete the same problem as Shaheen, simplifying first and list the restrictions.

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5. Multiply each expression. List the restrictions for the variables.

a. $\frac{3ab^2}{4c} \cdot \frac{2c^2}{27ab}$

b. $\frac{3x}{5x - 15} \cdot \frac{x - 3}{9x^2}$

c. $\frac{x + 5}{x^2 - 4x + 3} \cdot \frac{x - 3}{4x + 20}$

d. $\frac{7x - 7}{3x^2} \cdot \frac{x + 5}{9x^2 - 9} \cdot \frac{x^2 - 5x - 6}{x^3 + 6x^2 + 5x}$

6. Is the set of rational expressions closed under multiplication? Explain your reasoning.

PROBLEM 2 Topsy Turvy World

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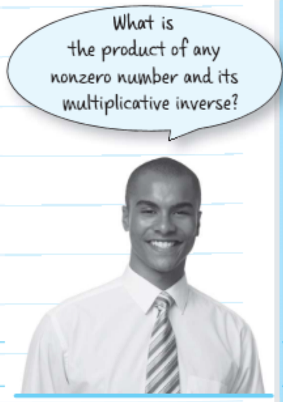
Dividing rational expressions uses the same process as dividing rational numbers.

Notice that when you multiply $\frac{4}{5}$ by a form of 1, in this case $\frac{3}{3}$, you maintain equivalent fractions:

$$\frac{4}{5}\cdot\frac{3}{3}=\frac{12}{15}$$

$$\frac{4}{5}=\frac{12}{15}$$



This same process works when the numerator and denominator are fractions. Consider $\frac{\frac{4}{5}}{\frac{3}{7}}$.

When you multiply by a form of 1, in this case $\frac{\frac{7}{3}}{\frac{7}{3}}$, you maintain equivalent fractions:

$$\frac{\frac{4}{5}\cdot\frac{7}{3}}{\frac{3}{7}\cdot\frac{7}{3}}=\frac{\frac{28}{15}}{\frac{21}{21}}=\frac{\frac{28}{15}}{1}=\frac{28}{15}$$

$$\frac{\frac{4}{5}}{\frac{3}{7}}=\frac{28}{15}$$

1. What is special about the form of 1 used to multiply $\frac{\frac{4}{5}}{\frac{3}{7}}$?

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You may recall that to divide fractions, you multiply by the multiplicative inverse of the divisor.

	Rational Numbers	Rational Expressions Involving Variables
Example 1	$\frac{1}{5} \div \frac{3}{10} = \frac{1}{\cancel{5}_1} \cdot \frac{\cancel{10}^2}{3}$ $= \frac{2}{3}$	$\frac{xy^2}{5z} \div \frac{3xy}{10z^2} = \frac{\cancel{xy^2}^{y}}{{}_1\cancel{5z}} \cdot \frac{\cancel{10z^2}^{2z}}{3\cancel{xy}}$ $= \frac{2yz}{3}; x, y, z \neq 0$
Example 2	$\frac{6}{7} \div 4 = \frac{\cancel{6}^3}{7} \cdot \frac{1}{\cancel{4}_2}$ $= \frac{3}{14}$	$\frac{6a^3}{7b} \div 4a = \frac{\cancel{6a^3}^{3a^2}}{7b} \cdot \frac{1}{\cancel{4a}_2}$ $= \frac{3a^2}{14b}; a, b \neq 0$

2. Analyze the examples shown in the table.

   a. Explain the process for dividing each expression.

   b. In Example 2, explain why $\frac{1}{4a}$ is the multiplicative inverse of $4a$.

3. Ranger calculates the quotient of $\frac{4x}{x^2y^2 - xy} \div \frac{x^2 - 4}{3x^2 + 19x - 14} \div \frac{x - 2}{xy}$.

> *Ranger*
>
> $$\frac{4x}{x^2y^2 - xy} \div \frac{x^2 - 4}{3x^2 + 19x - 14} \div \frac{x - 2}{xy} = \frac{4x}{x^2y^2 - xy} \cdot \frac{3x^2 + 19x - 14}{x^2 - 4} \cdot \frac{xy}{x - 2}$$
>
> $$= \frac{4\cancel{x}\cancel{(x - 1)}(x + 6)\cancel{(x + 4)}(x - 3)}{5\cancel{x^2}_{x}(x - 4)\cancel{(x + 4)}(x - 6)\cancel{(x - 1)}}$$
>
> $$= \frac{4x^2 + 12x - 72}{5x^3 - 50x^2 + 120x}$$

List the restrictions for the variables.

4. Determine the quotients of each expression.

a. $\frac{9ab^2}{4c} \div \frac{18c^2}{5ab}$

b. $\frac{7x^2}{3x^2 - 27} \div \frac{4x^2}{3x - 9}$

c. $\frac{3x^2 + 15x}{x^2 - 3x - 40} \div \frac{5x^2}{x^2 - 64}$

d. $\frac{4x}{x^2y^2 - xy} \div \frac{x^2 - 4}{3x^2 + 19x - 14} \div \frac{x - 2}{xy}$

5. Is the set of rational expressions closed under division? Explain your reasoning.

Be prepared to share your solutions and methods.